# Communications and Control Engineering

Springer
*London*
*Berlin*
*Heidelberg*
*New York*
*Barcelona*
*Hong Kong*
*Milan*
*Paris*
*Singapore*
*Tokyo*

*Published titles include:*

*Nonlinear Control Systems (3rd edition)*
Alberto Isidori

*Theory of Robot Control*
C. Canudas de Wit, B. Siciliano and G. Bastin (Eds)

*Fundamental Limitations in Filtering and Control*
María M. Seron, Julio Braslavsky and Graham C. Goodwin

*Constructive Nonlinear Control*
R. Sepulchre, M. Janković and P.V. Kokotović

*A Theory of Learning and Generalization*
M. Vidyasagar

*Adaptive Control*
I.D. Landau, R. Lozano and M.M'Saad

*Stabilization of Nonlinear Uncertain Systems*
Miroslav Krstić and Hua Deng

*Passivity-based Control of Euler-Lagrange Systems*
Romeo Ortega, Antonio Loría, Per Johan Nicklasson and Hebertt Sira-Ramírez

*Stability and Stabilization of Infinite Dimensional Systems with Applications*
Zheng-Hua Luo, Bao-Zhu Guo and Omer Morgul

*Nonsmooth Mechanics (2nd edition)*
Bernard Brogliato

*Nonlinear Control Systems II*
Alberto Isidori

*$L_2$-Gain and Passivity Techniques in Nonlinear Control*
Arjan van der Schaft

*Control of Linear Systems with Regulation and Input Constraints*
Ali Saberi, Anton A. Stoorvogel and Peddapullaiah Sannuti

*Robust and H∞ Control*
Ben M. Chen

*Computer Controlled Systems*
Efim N. Rosenwasser and Bernhard P. Lampe

*Dissipative Systems Analysis and Control*
*Rogelio Lozano, Bernard Brogliato, Olav Egeland and Bernhard Maschke*

*Control of Complex and Uncertain Systems*
Stanislav V. Emelyanov and Sergey K. Korovin

Ian R. Petersen, Valery A. Ugrinovskii
and Andrey V. Savkin

# Robust Control Design Using $H^{\infty}$ Methods

With 53 Figures

Springer

Ian R. Petersen, BE, MSEE, PhD
School of Electrical Engineering, ADFA, Canberra ACT 2600, Australia

Valery A. Ugrinovskii, PhD
School of Electrical Engineering, ADFA, Canberra ACT 2600, Australia

Andrey V. Savkin, PhD
Department of Electrical and Electonic Engineering, University of Western Australia, Nedlands,WA6907 Australia

*Series Editors*

E.D. Sontag • M. Thoma

ISBN 1-85233-171-2 Springer-Verlag London Berlin Heidelberg

British Library Cataloguing in Publication Data
Petersen, Ian R.
Robust control design using H- [infinity symbol] methods. -
(Communication and control engineering)
1.Automatic control 2. H [infinity symbol] control
I.Title II. Ugrinovskii, Valery A. III.Savkin, Audrey V.
629.8'312
ISBN 1852331712

Library of Congress Cataloging-in-Publication Data
Petersen, Ian Richard, 1956-
Robust control design using H-È methods / Ian R. Petersen, Valery A. Ugrinovskii, and Audrey V. Savkin.
p. cm. -- (Communication and control engineering series)
ISBN 1-85233-171-2 (alk. paper)
1. Robust control. I. Ugrinovskii, Valery A., 1960- II. Savkin, Audrey V. III. Tile. IV. Series.
TJ217.2P48 2000
629.8'312--dc21 99-056302


Printed in Great Britain



Typesetting: Camera ready by authors
Printed and bound by Athenæum Press Ltd., Gateshead, Tyne & Wear
69/3830-543210 Printed on acid-free paper SPIN 10713809

# Preface

One of the key ideas to emerge in the field of modern control is the use of optimization and optimal control theory to give a systematic procedure for the design of feedback control systems. For example, in the case of linear systems with full state measurements, the linear quadratic regulator (LQR) approach provides one of the most useful techniques for designing state feedback controllers; e.g., see [3]. Also, in the case of linear systems with partial information, the solution to the linear quadratic Gaussian (LQG) stochastic optimal control problem provides a useful technique for the design of multivariable output feedback controllers; see [3]. However, the above optimal control techniques suffer from a major disadvantage in that they do not provide a systematic means for addressing the issue of robustness. The robustness of a control system is a fundamental requirement in designing any feedback control system. This property of the control system reflects an ability of the system to maintain adequate performance and in particular, stability in the face of variations in the plant dynamics and errors in the plant model which is used for controller design. The enhancement of robustness is one of the main reasons for using feedback; e.g., see [87].

The lack of robustness which may result when designing a control system via standard optimal control methods, such as those mentioned above[1], has been a major motivation for research in the area of robust control. Research in this area has resulted in a large number of approaches being available to deal with the issue of robustness in control system design; e.g., see [47]. These methods include those

[1] It is known for example, that in the partial information case, an LQG controller may lead to very poor robustness properties; e.g., see [50].

based on Kharitonov's Theorem (see [104, 7]), $H^\infty$ control theory (e.g., see [52, 9]) and the quadratic stabilizability approach (e.g., see [6, 102]). Also, the theory of absolute stability should be mentioned; e.g., see [120, 2, 137, 162]. For the purposes of this book, it is worth mentioning two important features of absolute stability theory. These features lead us to consider absolute stability theory as a forerunner of contemporary robust control theory. Absolute stability theory was apparently one of the first areas of control theory that introduced and actively used the notion of an uncertain system. Also, absolute stability theory introduced and made use of matrix equations that were termed Lur'e equations in the Russian literature and are known as Riccati equations in the West; e.g., see [119, 95, 256, 23].

In this book, we present the reader with some of the latest developments in robust control theory for uncertain systems. In particular, we will consider the quadratic stabilizability approach to the control of uncertain systems with *norm bounded-uncertainty* and a more general absolute stabilizability approach to the control of uncertain systems described by *integral quadratic constraints (IQCs)*. The reader will see that the integral quadratic constraint concept and its generalizations leads to an uncertain system framework in which the standard LQR and LQG optimal controller design methodologies can be extended into minimax optimal control and guaranteed cost control methodologies. Such an uncertain system framework allows the designer to retain existing methods of LQR/LQG design while the issue of robustness can be addressed by an appropriate choice of the uncertainty structure in the uncertain system model. In the book, this approach is illustrated by the two practical problems, a missile autopilot design problem and an active noise control problem.[2]

Note that for the most part, the robust control problems considered in this book are closely related to corresponding $H^\infty$ control problems. Furthermore, throughout the book, the solutions to these robust control problems will be formulated in terms of Riccati equations of the form arising in $H^\infty$ control theory. These facts motivate the title of this book.

The book focuses only on robust control system design problems. Other related topics such as robust filtering and robust model validation are outside the scope of the book. However, the methods presented here have been applied successfully to these and other related problems; e.g., see [196, 186, 192, 130, 199, 201, 203, 132]. In particular, robust filtering problems were carefully studied in the research monograph [159].

Another area in which the methods and ideas developed in this book have been successfully applied is in the analysis and synthesis of hybrid control systems; see [204, 175, 205, 214]. A survey of results on hybrid control systems together with a bibliography relating to this area can be found in the book [122].

---

[2]The reader who is interested in the practical aspects of these examples can find some additional details on the book website http://routh.ee.adfa.edu.au/~irp/RCD/index.html.

It is worth noting that in this book, we deal only with continuous-time systems. This is in no way due to limitations in the theory. The reader who is interested in robust control of discrete-time uncertain systems can use this book as a conceptual guideline to the analogous discrete-time theory. Moreover, discrete-time counterparts to the majority of the results in this book have already been published. We will point out the relevant references in the text.

In the production of this book, the authors wish to acknowledge the support they have received from the Australian Research Council. Also the authors wish to acknowledge the major contributions made by their colleagues Duncan McFarlane, Matthew James, Paul Dupuis, Uffe Thygesen and Hemanshu Pota in the research that underlies much of the material presented in this book. Furthermore, the first author is grateful for the enormous support he has received from his wife Lynn and children Charlotte and Edward. Also, the second author is truly thankful to Grisha, Anna and Elena for their love, encouragement and emotional support.

# Contents

# Frequently used notation

$\mathbf{R}^n$ is the $n$-dimensional Euclidean space of real vectors.

$\mathbf{R}^k_+$ is the positive orthant of $\mathbf{R}^n$. That is, $\mathbf{R}^k_+$ consists of all vectors in $\mathbf{R}^n$ such that each component is positive.

$\mathrm{cl}\, M$ denotes the closure of a set $M \subset \mathbf{R}^n$.

$\|\cdot\|$ is the standard Euclidean norm.

$\|A\|$ where $A$ is a matrix, is the induced operator norm of $A$.

$A'$ denotes the transpose of the matrix $A$.

$\rho(A)$ denotes the spectral radius of a matrix $A$.

$\lambda_{max}(N)$ denotes the maximum eigenvalue of a symmetric matrix $N$.

$\mathrm{tr}\, N$ denotes the trace of a matrix $N$.

$M \geq N$ ($M > N$) where $M$ and $N$ are square matrices, denotes the fact that the matrix $M - N$ is non-negative (positive) definite.

$C(\mathcal{J}, X)$ is the set of continuous functions $f \colon \mathcal{J} \to X$.

$C_b(\mathbf{R}^n)$ denotes the set of bounded continuous functions from $\mathbf{R}^n$ to $\mathbf{R}$.

$\mathbf{L}_2[T_1, T_2)$ is the Lebesgue space of square integrable functions, defined on $[T_1, T_2)$, $T_1 < T_2 \leq \infty$.

$\|x(\cdot)\|_2^2$ is the norm in $\mathbf{L}_2[T_1, T_2)$, that is $\|x(\cdot)\|_2^2 = \int_{T_1}^{T_2} \|x(t)\|^2 dt$, $T_1 < T_2 \leq \infty$.

$\langle x, y \rangle$ is the inner product in a Hilbert space.

$G^*$ denotes the adjoint of an operator $G$.

$H^\infty$ is the Hardy space of complex functions $f(s)$ analytic and bounded in the open right half of the complex plane.

$\|f(s)\|_\infty$ is the norm in the Hardy space $H^\infty$, $\|f(s)\|_\infty := \sup_{\mathrm{Re}\, s>0} |f(s)|$.

$\mathcal{P}$ is a set of uncertainty inputs

$\Xi$ is a set of admissible uncertainty inputs.

$\delta \downarrow 0$ denotes the fact that $\delta$ approaches zero monotonically from above. Similarly, $T \uparrow \infty$ means that $T$ approaches $\infty$ monotonically.

$(\Omega, \mathcal{F})$ is a measurable space.

$W(t)$ is a Wiener process.

$\mathbf{E}^Q$ is the expectation operator with respect to a probability measure $Q$. Also, $\mathbf{E}$ is the expectation with respect to a reference probability measure.

a.s. is an acronym for *almost surely*. In probability theory, this refers to a property which is true for almost all $\omega \in \Omega$, *i.e.*, the set $\{\omega :$ the property is false$\}$ has probability zero.

$Q \ll P$ denotes the fact that the measure $Q$ is absolutely continuous with respect to the measure $P$ where $Q$ and $P$ are Lebesgue measures defined on a measurable space.

$\mathbf{L}_p(\Omega, \mathcal{F}, Q)$ is the Lebesgue space of $\mathcal{F}$-measurable functions $f: \Omega \to X$ such that $\mathbf{E}^Q \|f\|_X^p < \infty$; here $X$ is a Banach space.

$\mathbf{L}_2(s, T; \mathbf{R}^n)$ is the Hilbert space generated by the $(t, \omega)$-measurable non-anticipating random processes $x(t, \omega) : [s, T] \times \Omega \to \mathbf{R}^n$.

$|||\cdot|||$ is the norm on the space $\mathbf{L}_2(s, T; \mathbf{R}^n)$ defined by $|||\cdot||| = \left(\int_s^T \mathbf{E}\|\cdot\|^2 dt\right)^{1/2}$.

$a \wedge b$ denotes the minimum of the two real numbers $a$ and $b$.

$\mathcal{M}(\mathbf{R}^n)$ denotes the set of probability measures on the set $\mathbf{R}^n$.

# 1. Introduction

## 1.1 The concept of an uncertain system

In designing a robust control system, one must specify the class of uncertainties the control system is to be robust against. Within the modern control framework, one approach to designing robust control systems is to begin with a plant model which not only models the nominal plant behavior but also models the type of uncertainties which are expected. Such a plant model is referred to as an uncertain system.

There are many different types of uncertain system model and the form of model to be used depends on type of uncertainty expected and the tractability of robust control problem corresponding to this uncertain system model. In many cases, it is useful to enlarge the class of uncertainties in the uncertain system model in order to obtain a tractable control system design problem. This process may however lead to a conservative control system design. Thus, much of robust control theory can be related to a trade off between the conservatism of the uncertain system model used and the tractability of the corresponding robustness analysis and robust controller synthesis problems.

Some commonly occurring uncertainty descriptions are as follows:

(i) A constant or time varying real parameter representing uncertainty in the value of a parameter in the system model; e.g., uncertainty in a resistance value in an electrical circuit.

(ii) A transfer function representing the uncertainty which might arise from neglecting some of the system dynamics; e.g., the effect of neglecting parasitic capacitances in an electrical circuit.

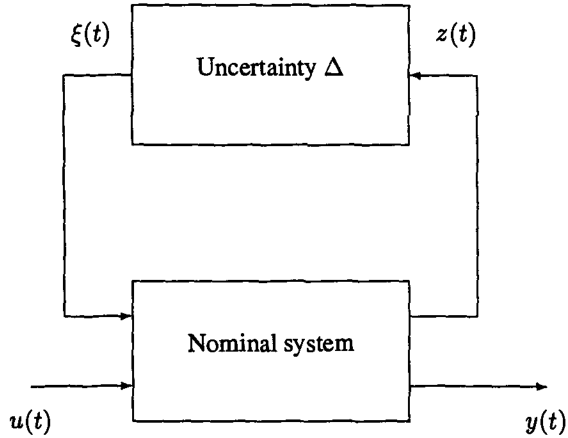


Figure 1.1.1. Uncertain system model block diagram

(iii) A nonlinear mapping which represents the uncertainty due to neglected nonlinearities.

An important class of uncertain system models involves separating the nominal system model from the uncertainty in the system in a feedback interconnection; see Figure 1.1.1. Such a feedback interconnection between the nominal model and the uncertainty is sometimes referred to as an linear fractional transformation (LFT); e.g., see [48].

Whatever the form of the uncertainty $\Delta$ in an uncertain system, it is typically a quantity which is unknown but bounded in magnitude in some way. That is, we do not know the value of the uncertainty but we know how big it can be. Some typical uncertainty bounds are as follows:

(i) A real time-varying uncertain parameter $\Delta(t)$, can be bounded in absolute value:

$$|\Delta(t)| \leq \mu \quad \text{for all } t \geq 0. \tag{1.1.1}$$

This includes time-varying norm-bounded uncertainty blocks; e.g., see [102].

(ii) If $\Delta(s)$ is an uncertain transfer function, we could bound its magnitude at all frequencies:

$$|\Delta(j\omega)| \leq \mu \quad \text{for all } \omega. \tag{1.1.2}$$

This amounts to a bound on the $H^\infty$ norm of the transfer function $\Delta(s)$; e.g., see [51].

Also, note that each of the above uncertainty bounds can be extended in a straightforward way to a matrix of uncertainties.

Uncertain systems of the types described above have been widely studied. Furthermore, useful solutions have been obtained to various robust control problems for these classes of uncertain systems; e.g., see [150, 142, 102, 156, 157, 146, 51, 58]. However, these classes of uncertain systems do have some deficiencies.

Consider the class of systems with norm bounded uncertainties of the form (1.1.1). Although this class of uncertain systems allows for time-varying uncertain parameters and also for cone-bounded memoryless nonlinearities, it does not allow for dynamic uncertainties such as might arise from unmodeled dynamics. Another problem concerns the results which can be obtained for this class of uncertainties. Necessary and sufficient conditions for quadratic stabilizability of such systems can be obtained. However, the notion of quadratic stabilizability does not directly relate to the behavior of the system. Also, although tight state-feedback guaranteed cost results can be obtained (see [156, 157] and Section 5.2 of this book), the generalization to the output-feedback case is less satisfactory and only leads to upper bounds. All of these facts motivate the integral quadratic constraint (IQC) uncertainty description which will be described in the sequel.

$H^\infty$ type uncertainty constraints of the form (1.1.2) provide further motivation for an uncertainty description in terms of integral quadratic constraints. First consider the transfer function uncertainty block in Figure 1.1.1. From the definition of the norm on the Hardy space $H^\infty$, it follows that the frequency domain bound (1.1.2) is equivalent to the *frequency domain* integral quadratic constraint

$$\int_{-\infty}^{\infty} |\hat{\xi}(j\omega)|^2 d\omega \leq \int_{-\infty}^{\infty} |\hat{z}(j\omega)|^2 d\omega \tag{1.1.3}$$

which must be satisfied for all signals $z(t)$ provided these integrals exist. Here, $\hat{\xi}(s)$ is the Laplace transforms of the uncertainty input $\xi(t)$. Also, $\hat{z}(s)$ is the Laplace transform of the uncertainty output $z(t)$. Furthermore, $z(t)$ is the input to the uncertainty block $\Delta$ and $\xi(t)$ is the output of this uncertainty block.

It is well known (e.g., see [124]) that integral quadratic constraints of the form of (1.1.3) may be applied in the analysis of nonlinear systems. In particular, a number of the system properties such as passivity, structural information about perturbations, energy constraints on exogenous perturbations, saturations, uncertain delays etc., allow for a description in terms of suitable constraints of the form (1.1.3). For further details, we refer the reader to the frequency domain integral quadratic constraints given in [124]. However as mentioned in [124], there is an evident problem in using this form of uncertainty description. This is because the condition (1.1.3) makes sense only if the signals $\xi(t)$ and $z(t)$ are square integrable. Reference [124] points out how this problem can be resolved in certain situations. For example, in the case where the nominal system is stable, this problem can be easily overcome. However in stabilization problems, it is possible that the uncontrolled system is *unstable* and then its inputs and outputs will not be square summable unless the system is controlled using a stabilizing controller. In this book, we show that the problems arising from uncertain systems with an unstable nominal can be overcome to a large extent by using *time domain versions* of the integral quadratic constraint uncertainty description.

An uncertain system model must be chosen so that it captures the essential features of the real system and the uncertainty in that system. Also, the uncertainty class must be chosen so that it leads to a tractable solution to the control problem under consideration. These facts have led us to use the time domain integral quadratic constraint uncertainty descriptions presented in Section 2. Since this form of uncertainty description prevails throughout the book, from now on we will usually drop the words "time domain". That is, we will use the expression *Integral Quadratic Constraint (IQC)* to denote time domain integral quadratic constraints.

There are a number of advantages in dealing with uncertain systems defined by IQCs. The class of uncertainties satisfying an IQC is richer than the class of uncertainties satisfying the corresponding norm bound condition. Also, a time domain IQC can be applied to model uncertainty in finite-horizon problems. Among other advantages of the time domain IQC uncertainty description, we note that this uncertainty description allows us to model structured uncertain dynamics in systems subject to stochastic noise processes. Our motivation for considering uncertain systems which are subject to stochastic noise process disturbances is twofold. In the first instance, many engineering control problems involve dealing with systems which are subject to disturbances and measurement noise which can be well modeled by stochastic processes. A second motivation is that in going from the state-feedback linear quadratic regular (LQR) optimal control problem to the measurement feedback linear quadratic Gaussian (LQG) optimal control problem, a critical change in the model is the introduction of noise disturbances. Hence, it would be expected that in order to obtain a reasonable generalization of LQG control for uncertain systems, it would be necessary to consider uncertain systems which are subject to disturbances in the form of stochastic noise processes.

Note that stochastic extensions to the integral quadratic constraint uncertainty description provide a possible approach to the problem of non-worst case robust controller design. Although the worst case design methodology has proved its efficacy in various engineering problems, it suffers from the disadvantage that the designer lacks the opportunity to discriminate between "expected" uncertainties and those uncertainties which seldom occur. Indeed, the standard deterministic worst case approach to robust controller design presumes that all uncertainties are equally likely. That is, one can think of the uncertainty as arising from a uniformly distributed random variable taking its values in the space of uncertainties. However, this may not accurately represent the uncertainty in the system under consideration. For example, it may have been determined that the uncertainties have a probability distribution other than uniform and the designer may wish to make use of this *à priori* information about the distribution of the uncertainties. In this case, it is useful to consider a stochastic uncertain system in which the uncertainty has a stochastic nature. Hence, a suitable description of stochastic uncertainty is required. To this end, in this book we consider stochastic integral quadratic constraints or more generally, stochastic relative entropy constraints.