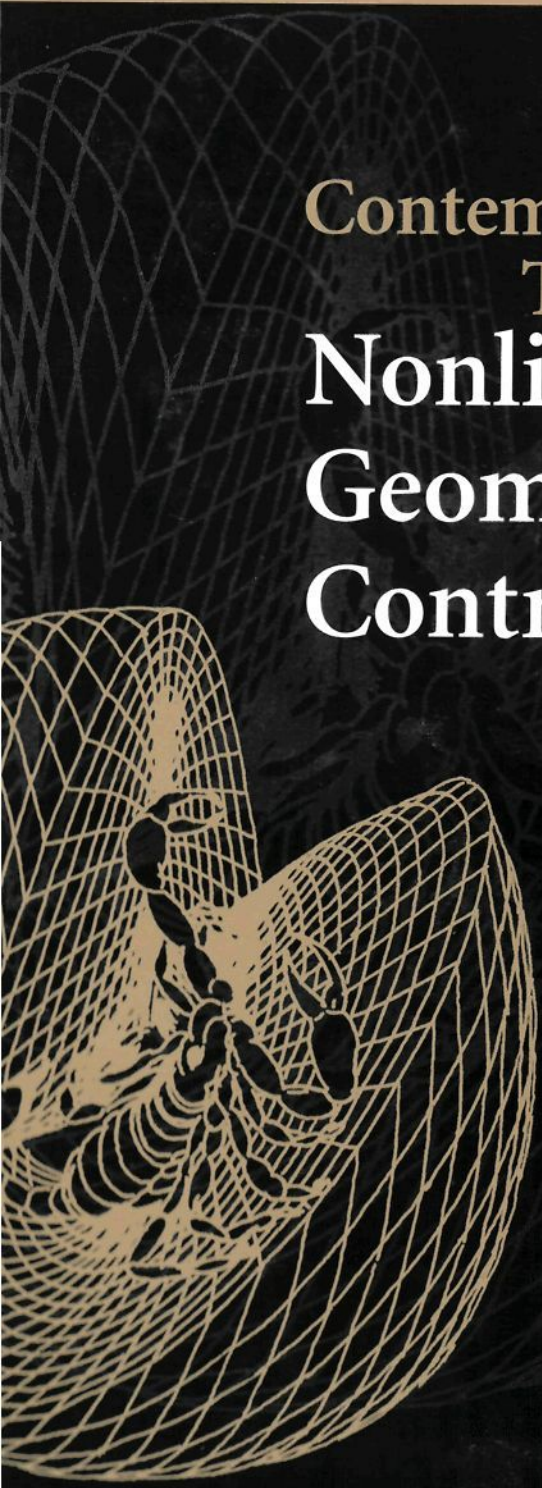
Contemporary Trends in Nonlinear Geometric Control Theory and its Applications



A. Anzaldo-Meneses
B. Bonnard
J.P. Gauthier
F. Monroy-Pérez

World Scientific

Contemporary Trends in Nonlinear Geometric Control Theory and its Applications

This page is intentionally left blank

Contemporary Trends in Nonlinear Geometric Control Theory and its Applications

A. Anzaldo-Meneses
(Universidad Autonoma Metropolitana-Azcapotzalco, México)

B. Bonnard
(Université de Bourgogne, France)

J.P. Gauthier
(Université de Bourgogne, France)

F. Monroy-Pérez
(Universidad Autonoma Metropolitana-Azcapotzalco, México)

World Scientific
New Jersey • London • Singapore • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

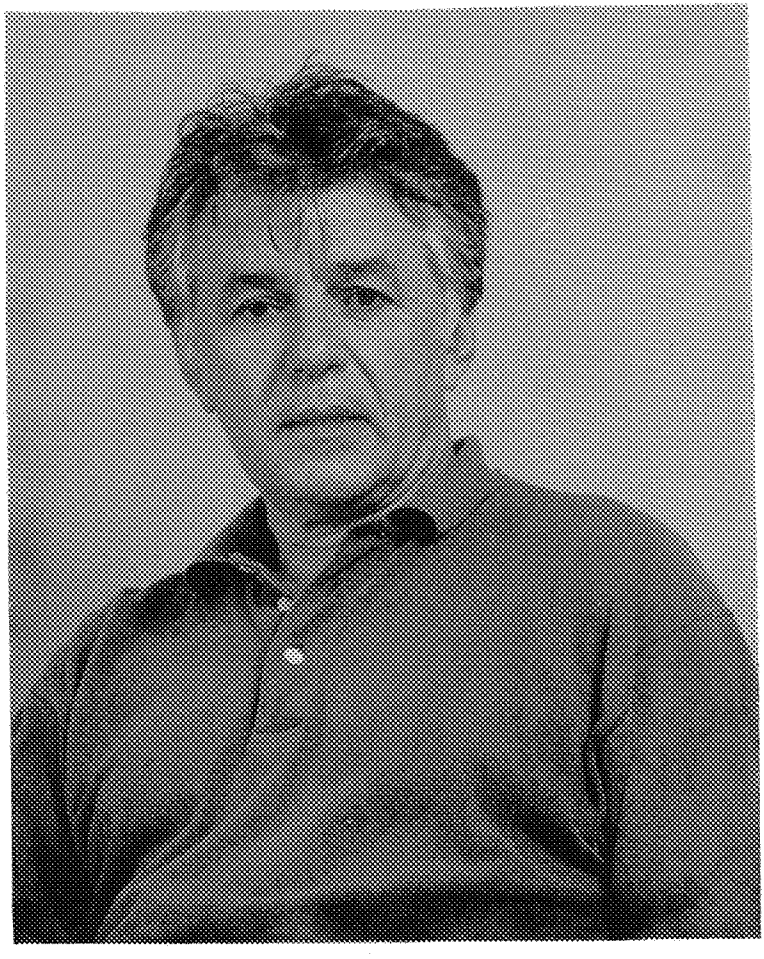
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

CONTEMPORARY TRENDS IN NONLINEAR GEOMETRIC CONTROL THEORY AND ITS APPLICATIONS

ISBN 981-02-4841-5

Printed in Singapore by World Scientific Printers (S) Pte Ltd



Velimir Jurdjevic

This page is intentionally left blank

Contents

This page is intentionally left blank

Foreword

Mathematical control theory has evolved from the study of practical problems in engineering and sciences to the elaboration of deep important concepts in mathematics and applied sciences. It shares with some other branches of modern applied mathematics the intense use of a broad range of mathematical tools which provides interesting challenges and possibilities for interactions with well established areas of pure mathematics.

The present volume is about contemporary trends in non-linear geometric control theory and its applications. It provides a fine collection of chapters in which new results, relevant open problems and important applications on academic and real-world problems are presented. Different topics in modern control theory are touched in the book, from fundamental theoretical issues to very practical applications. The contributions are intended to be self-contained and mutually independent, although there are several unifying viewpoints permeating all over the book, among others: a highly geometric point of view, a modern approach of control systems as families of vector fields, an extensive use of the Lie-algebra theoretical framework.

Some of the chapters in the present volume are chapters in extenso written on topics presented in lectures given during the international conference "Geometric Control Theory and applications" that took place in México City, in the friendly atmosphere of the UAM-Azcapotzalco in the summer of year 2000.

The conference celebrated also the *60th birthday of Velimir Jurdjevic* and this volume is dedicated to him. The scientific activity of Vel Jurdjevic as been influential in the research work of many of the authors in the book. His intellectual integrity and his innovative approach of control systems is present all over the book, from the theoretical contributions through the ones on applications.

Since the early seventies Vel Jurdjevic has written various seminal chapters in modern control theory, with some other authors he developed a set of innovative ideas on the controllability of non-linear control systems, extending the classical tools already known in linear control theory and uncovered the relevance of studying control systems on Lie groups, as a natural extension of control systems in linear spaces. Within this set of ideas Vel Jurdjevic co-authored a paper which has been pointed out among the most influential chapters of the century in modern control theory.

In the eighties Vel Jurdjevic together with some other authors paved the way for the understanding of control systems in Lie groups, exploiting the structure theory of Lie groups and Lie algebras to develop new theoretical tools in geometric no-linear control theory, such as the concepts of Lie determined systems and the Lie saturate of a family of vector fields. Vel Jurdjevic co-authored a number of papers were families of vector fields in semi-simple Lie groups and direct products of Lie groups were extensively studied. The classes of affine and polynomial systems were also studied within this formalism.

Since the early nineties Vel Jurdjevic has dedicated his attention to problems in geometry and mechanics, merging creatively classical ideas with those in modern optimal control theory. Several non-holonomic mechanical problems such as mechanical tops, plate-and-balls, etc., and geometric problems such as frame curves, elastic curves, constrained shortest paths, etc., have been studied by Vel Jurdjevic by means of the Maximum Principle and the systematic use of the symplectic geometry and the Hamiltonian formalism. The theoretical framework developed by Vel Jurdjevic in the understanding of all these problems includes the study of integrability theory of Hamiltonian systems and sub-Riemannian geometry in homogeneous spaces.

This book is dedicated to Vel Jurdjevic and contains a number of invited chapters written specially for the occasion by colleagues and friends of Vel Jurdjevic, all of them leading applied mathematicians and control theorists. There is also place for surveys on topics of current research which provide to the reader with a fresh presentation of the state of the art of modern geometric control theory. The volume touches different topics in modern control theory, it presents in an amenable style, fundamental theoretical issues as well as very practical applications. The reader will find an invaluable source of current research in this branch of applied mathematics.

The scientific activity in the geometric control theory community has reached a high level of intellectual maturity which is reflected in the merging of different branches of mathematics and science engineering. The chapters in this book are very good examples of this fruitful interactions. An intense and creative activity of mathematical control theorists has brought innovative approaches in the understanding and the solution of practical and theoretical problems. Particularly, geometric techniques have been successfully applied in different aspects of the theory of non-linear control systems and its applications.

The volume contains important new results obtained by a consistent and

creative use of various geometric techniques on different topics of non-linear control systems. All the results presented along the chapters of this volume have been obtained using the powerful machinery of differential geometry, algebraic geometry, symplectic geometry, and the geometric theory of dynamical systems.

The book has two parts, the first part contains four contributions which survey on various themes of non-linear geometric control theory written by very well known leading specialists. The second part contains contributed chapters on different topics on non-linear geometric control theory and its applications.

Controllability of non-linear systems is a concept which is intrinsically related with the Lie algebra associated to it, different results in this respect are presented in the book.

The survey written by J. Lawson and D. Mittenhuber discusses the topic by exploiting the structure theory of Lie groups and Lie semi-groups, whereas in the contribution by J. Cariñena and A. Ramos, the topic is developed in a dynamical systems approach presenting a generalization of the method proposed by Wei and Norman. Controllability is also studied in connection with optimal control theory and with the use of coordinates of first kind in the chapters by A. Jourani and M. Kawski, respectively.

Reachability, observability and linearization of non-linear control systems are notions rooted in the very foundations of mathematical control theory, different approaches of these topics are presented in the volume.

The contribution by R.M. Bianchini discusses reachability by means of geometric objects such as tangent cones and considering special types of variations. Observability is touched in the chapter written by S. Diop under a differential algebraic point of view. In the contribution written by E.Busvelle and J.P. Gauthier the construction of the so-called high-gain observers is studied with a revisited Kalman filtering technique. A novel approach on linearization of control systems is presented in the chapter by L. Baratchat M. Chyba and J.B. Pomet extending the notion of equivalence by means of techniques of topological dynamics.

Optimal control theory might be thought as the modern continuation of the classical calculus of variations, which has been an archetypical way of thinking in mathematics and physics. This solid mathematical discipline

finds in modern geometric optimal control theory a new stage for its development and a continuation of a solid intellectual tradition. Moreover, the prime ideas of optimal control theory go deep into the actual development of sub-Riemannian geometry, and the understanding of important geometrical objects such as caustics, small radius balls and wave fronts. Optimal control theory and calculus of variations are discussed along the volume from different points of view.

The survey by V. Jurdjevic and F. Monroy-Pérez treats variational problems in Lie groups taking the topic further, from the study of elastic curves to the investigation of certain aspects of the integrability theory of dynamical systems, which that has been always an important source of ideas in mathematical physics and algebraic geometry. The survey by H.J. Sussmann and J.C. Willems provides an authorized historical recollection of calculus of variations into the perspective of modern optimal control theory. The authors argue that modern control theory provides the best and mathematically most natural setting to do justice to Johann Bernoulli's famous brachistochrone problem.

Symplectic geometry provides the right setting for the invariant formulation of the Maximum Principle which allows the study of strong optimality as seen in the contribution by A. Agrachev, G. Stefani and P. Zezza. Optimal control problems are discussed within the ideas of sub-Riemannian geometry in the chapter written by A. Anzaldo-Meneses and F. Monroy-Pérez and that written by E. Trelat.

A more classical approach of the calculus of variations is taking in the contribution by N. Nuñez-Yepez, J. Delgado and A. Salas-Brito in which symmetries and constants of motion for the Jacobi equation are discussed.

The theory of distributions and its classification has been an important field of study in differential geometry. Since the early days of geometric control theory this machinery has been used and vice versa control theoretical techniques have produced interesting new results in the theory of distributions. The survey by W. Respondek and W. Pasillas-Lépine presents the state of the art for the contact case.

Applications are all over the book, from academic problems to the very practical real-world applications. The optimal control of the atmospheric arc of the space shuttle is discussed in chapter by B. Bonnard, E. Busvelle and G. Launay. The hover-craft vessel is studied in the contribution by H. Sira-

Ramirez by means of sliding mode control techniques. Some aspects of the electrodynamics of non-relativistic particles are treated in the chapter by A. Anzaldo-Meneses and F. Monroy-Pérez. And the distillation column problem is presented in the contribution by E. Busvelle and J.P. Gauthier within the theory of observability of nonlinear control systems.

The volume contains several new mathematical ideas generated by geometric control theory techniques, which open new directions of research beyond of control theory. The chapter by B. Jackubcyk discusses a very interesting connection of the theory of realization of non-linear control systems and partial differential equations. Whereas the contribution by D.Varolin introduces control theoretic ideas in the theory of holomorphic vector fields.

We thank all the authors for their contributions to this volume. We thank also the CONACYT, México, the CNRS, France and the Universidad Autónoma Metropolitana-Azcapotzalco, for their important support to carry out this project.

THE EDITORS

A. Anzaldo-Meneses
B. Bonnard
J.P. Gauthier
F. Monroy-Pérez

Part I

Invited Survey Chapters

This page is intentionally left blank