# Quantum Noise in Mesoscopic Physics

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# Quantum Noise in Mesoscopic Physics

edited by

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Springer-Science+Business Media, B.V.

Proceedings of the NATO Advanced Research Workshop on
Quantum Noise in Mesoscopic Physics
Delft, The Netherlands
2–4 June 2002

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-1-4020-1240-2 ISBN 978-94-010-0089-5 (eBook)
DOI 10.1007/978-94-010-0089-5

*Printed on acid-free paper*

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# Preface

This book is written to conclude the NATO Advanced Research Workshop "Quantum Noise in Mesoscopic Physics" held in Delft, the Netherlands, on June 2-4, 2002. The workshop was co-directed by M. Reznikov of Israel Institute of Technology, and me. The members of the organizing committee were Yaroslav Blanter (Delft), Chirstopher Glattli (Saclay and ENS Paris) and R. Schoelkopf (Yale). The workshop was very successful, and we hope that the reader will be satisfied with the scientific level of the present book. Before addressing scientific issues I find it suitable to address several non-scientific ones.

The workshop was attended by researchers from many countries. Most of them perform their activities in academic institutions, where one usually finds the necessary isolation from the problems and sores of the modern world. However, there was a large group of participants for which such isolation was far from perfect. War, hatred, and violence rage just several miles away of their campuses and laboratories, poisoning everyday life in the land of Israel.

Science and scientists can hardly help to resolve the situation. Albeit there is something we can do. We witness various actions that differ much in means but have a common goal: to undermine scientific and cultural ties between Israel and the rest of the world. Just two examples. Before the workshop, on April 6, 2002, 120 university professors published a letter in *Gardian* calling a moratorium on partnership of Israel in EU projects. After the workshop, on July 31, 2002, a bomb hidden in a handbag exploded in a crowded cafeteria at Hebrew University in Jerusalem. Four foreign exchange students were among the fallen. These actions can and must be confronted. That is why our workshop was in partnership with Israel, and we were proud to profit from a traditionally high level of Israeli research in the field of quantum noise.

We shall remain firm and hopeful, and we overcome. To support this with an example, I recall my first scientific discussion with the Israeli co-director of the present workshop, Michael Reznikov. It took place in 1981 in a Soviet military training facility. At that time, neither the circumstances of our life nor the general political situation inspired us to do research. We were trained to contribute to a large-scale nuclear weapons game, and our future looked dim if not apocalyptic. Yet the Almighty was merciful to all people, and us, so that the evil was defused. One could doubt the efficiency and the moral foundations of the North Atlantic Treaty Organization. However, it was one of the means, which defused the evil. In 1981 we were imaginative and ambitious young people. However, we could not think of co-directing a scientific conference supported by NATO. This fact from the past enormously enhances our appreciation of NATO support.

I would like to conclude by acknowledging other persons and organizations. The workshop would not take place without Yaroslav Blanter, who generously invested his time, energy and enthusiasm into the venture. Christian Glattli and Rob

Schoelkopf contributed much with their advice and organizational efforts. The financial support of Royal Netherlands Academy of Arts and Sciences (KNAW) and Foundation for Fundamental Research on Matter (FOM) is gladly appreciated. Delft University of Technology was so kind as to serve our debts. Yvonne Zwang was always ready to assist us without asking for reimbursement.

Last but not least, I would like to thank all the participants of the workshop and contributors of this volume. Your enthusiastic response exceeded the expectations of the organizers. This is the best award for our activities.

Yuli V. Nazarov,
Delft University of Technology

# Introduction

The field of quantum noise in mesoscopic physics has been intensively developing for more than a decade. Remarkably, the developments do not seem to slow down yet. This book presents a collection of mini-reviews where the leading research teams summarize the most recent results and chart new directions. I was pleasantly surprised by enthusiasm of contributors who were willing to invest their time and energy in writing. This shows that the book is timely. Taken together, the reviews give a fairy representative snapshot of the modern state of the field. This state is still not coherent; an attentive reader will notice not only different approaches and conflicting views but contradictory results and interpretations as well. The contradictions should be present in the book since they drive the rapid evolution of the field. A general reader would possibly be more interested in driving forces and high intellectual content than in concrete results.

The book is divided into three parts: shot noise, quantum measurement and entanglement, and full counting statistics. These are the three main research streams. Since many contributions are simultaneously related to two or three streams, their placing may be rather subjective.

The whole field was pioneered in late eighties by Lesovik, Büttiker, Beenakker and Levitov; I am happy that two of the four could contribute to the book. The shot noise part opens with an article of Büttiker that presents a broad study of the sign of noise correlation and its (un)relation to Fermi statistics. Two subsequent contributions illustrate all the power and beauty of Landauer-Büttiker scattering approach. Van Ruitenbeek introduces atomic-size contacts as a brilliant experimental realization of a few mode scatterer. Not only PIN-code of transmission eigenvalues can be measured experimentally, the PIN-code works revealing harmony of noise properties even in such complicated situations as multiple Andreev reflection ( A. Martin-Rodero et al.)! The scattering becomes much more complicated in realistic diffusive conductors, this is revealed by Andreev reflection from near superconductors. High-frequency noise measurements (Reulet et al.) pinpoint these coherent effects, quantum circuit theory is required to reach a harmony between experiment and theory and calls on the full counting statistics. The theory of Bezuglyi et al. and the experiment of Strunk and Schönenberger pertain the incoherent multiple Andreev reflection regime that occur in longer and hotter NS structures and considerably enhances the noise. Glattli et al. present their pioneering experimental observation of non-transport photo-assisted partition noise. The contribution of Sukhorukov et al. concerns strongly interacting Coulomb blockade systems and reveals unexpected deviations of cotunneling noise from Poisson statistics.

The words "quantum measurement" and "entanglement" used to sound very abstract just a few years ago. The attempts to realize quantum manipulation at meso- and nanoscale have put these topics in the focus of practical research. It was soon recognized that the progress in quantum manipulation is impossible without deepening our knowledge about quantum noise that hinders the manipulation and affects the measurement. It was also recognized that a noise measurement may provide indispensable information concerning the entanglement, and the result of the manipulation. This initiated a new and most active research direction in the field. Yale collaboration presents a qubit as an ideal spectrum analyzer for quantum noise, their contribution being a fairly complete experimental proposal. Korotkov explicates the continuous measurement of a single qubit. His contribution would heal many from the plague of the field: making (carriers in) physics from semantic traps that arise from (mis)interpretation of quantum mechanics. The contribution of Averin possesses similar healing properties: linear measurement is explained. Egues et al. present a detailed description of the last crusade of Basel group, this is aimed to demonstrate the use of shot noise for detection of spin entanglement and polarization. Oliver at al. review their recent experiments in generating and detection of electron entanglement. Gavish et al. address the measurement of excess noise under conditions of amplification. Martin et al. propose experimental check of Bell inequalities for electrons by means of noise correlation measurement. Johanssson et al. investigate the feasibility of Single Electron Transistor for measuring qubits, this includes careful analysis of SET noise and back-action. Shnirman and Schön review dephasing and renormalization in a qubit placed into a dissipative environment.

I have felt in love with the full counting statistics, and probably would not be objective describing this research stream. My excuse is that I share this love with rapidly increasing number of people, some even seemed too prominent to experience such feelings. So that, full counting statistics is everything for us. It provides ultimate knowledge about charge transfer, it appears to be a solid foundation of the whole quantum transport, it may provide ultimate information concerning muli-particle entanglement ... I wished the first experiment in FCS was presented in the book. Unfortunately, it will appear elsewhere. Levitov, the founder of the FCS, introduces the FCS and reviews his achievements. Klich presents a novel derivation of Levitov's formula, this short contribution being pedagogically indispensable. Kindermann and me begin with a general analysis of quantum measurement aspects of the FCS. This quickly brings us to practical description of the FCS in electric circuits. Bagrets and me address the FCS in multi-terminal circuits treating the limits of non-interacting and Coulomb-blockaded electrons. Belzig discusses FCS for Andreev reflection. Gutman et al. review FCS of non-equilibrium electrons.

I hope that this book can serve as a good introduction to the field, both for a novice and an expert.

# PART ONE

## Short Noise

# REVERSING THE SIGN OF CURRENT-CURRENT CORRELATIONS

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## 1. Introduction

Dynamic fluctuation properties of mesoscopic electrical conductors provide additional information not obtainable through conductance measurement. Indeed, over the last decade, experimental and theoretical investigations of current fluctuations have successfully developed into an important subfield of mesoscopic physics. A detailed report of this development is presented in the review by Blanter and Büttiker [1].

In this work we are concerned with the correlation of current fluctuations which can be measured at different terminals of multiprobe conductors. Of particular interest are situations where, as a function of an externally controlled parameter, the sign of the correlation function can be reversed.

Electrical correlations can be viewed as the Fermionic analog of the Bosonic intensity-intensity correlations measured in optical experiments. In a famous astronomical experiment Hanbury Brown and Twiss demonstrated that intensity-intensity correlations of the light of a star can be used to determine its diameter [2]. In subsequent laboratory experiments of light split by a half-silvered mirror statistical properties of light were further analyzed [3]. Much of modern optics derives its power from the analysis of correlations of entangled optical photon pairs generated by non-linear down conversion [4]. The intensity-intensity correlations of a thermal Bosonic source are positive due to statistical bunching. In contrast, anti-bunching of a Fermionic system leads to negative correlations [5].

Concern with current-current correlations in mesoscopic conductors originated with Refs. [6, 7]. The aim of this work was to investigate the fluctuations and correlations for an arbitrary multiprobe conductor for which the conductance matrix can be expressed with the help of the scattering matrix [8, 9]. Refs. [6, 7] provided an extension of the discussions of shot noise by Khlus [10] and Lesovik [11] which applies to two-terminal conductors. These authors assumed from the outset that the transmission matrix is diagonal and provided expressions for the



*Y. V. Nazarov (ed.), Quantum Noise in Mesoscopic Physics,* 3–31.

two terminal shot noise in terms of transmission probabilities. It turns out that even for two probe conductors, shot noise can be expressed in terms of transmission probabilities only in a special basis (eigen channels). Such a special basis does not exist for multiprobe conductors and we are necessarily left with expressions for shot noise in terms of quartic products of scattering matrices [7, 12]. There are exceptions to this rule: for instance correlations in three-terminal one-channel conductors can also be expressed in terms of transmission probabilities only [13].

The reason that shot noise, in contrast to conductance, is in general not simply determined by transmission probabilities is the following: if carriers incident from different reservoirs (contacts) or quantum channels can be scattered into the same final reservoir or quantum channel, quantum mechanics demands that we treat these particles as indistinguishable. We are not allowed to be able to distinguish from which initial contact or quantum channel a carrier has arrived. The noise expressions must be invariant under the exchange of the initial channels [14–18]. The occurrence of exchange terms is what permitted Hanbury Brown and Twiss to measure the diameter of the stars: Light emitted by widely separated portions of the star nevertheless exhibits (a second order) interference effect in intensity-intensity correlations [2].

Experiments which investigate current-correlations in mesoscopic conductors have come along only recently. Oliver et al. used a geometry in which a "half-silvered mirror" is implemented with the help of a gate that creates a partially transparent barrier [19]. Henny et al. [20] separated transmission and reflection along edge states of a quantum point contact subject to a high magnetic field. In the zero temperature limit an electron reservoir compactly fills all the states incident on the conductor. A subsequent experiment by Oberholzer at al. [21] uses a configuration with two quantum point contacts, as shown in Fig. 1. This geometry permits to thin out the occupation in the incident electron beam and thus allows to investigate the transition in the correlation as we pass from degenerate Fermi statistics to dilute Maxwell-Boltzmann satistics. Anti-bunching effects vanish in the Maxwell-Boltzmann limit and the current-current correlation tends to zero as the occupation of the incident beam is diminished. The fact that in electrical conductors the incident beam is highly degenerate is what made these Hanbury Brown and Twiss experiments possible. In contrast, an emission of electrons into the vacuum generates an electron beam with only a feeble occupation of electrons [22] and for this reason an experiment in vacuum has in fact just been achieved only very recently [23]. Below we will discuss the experiments in electrical conductors in more detail.

Within the scattering approach, in the white noise limit, it can be demonstrated, that current-current correlations are negative, irrespective of the voltages applied to the conductor, temperature and geometry of the conductor [7, 12]. The wide applicability of this statement might give the impression, that in systems of Fermions current correlations are always negative. However, the proof

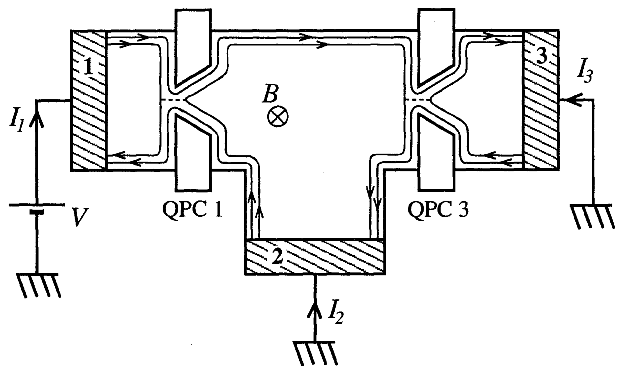


*Figure 1.* Experimental arrangement of Oberholzer at al. Current is injected at contact 1. One edge channel is perfectly transmitted (and noiseless) the other is partially transmitted at QPC 1 with probability $T_1$ and partially transmitted at QPC 3 with probability $T_3$ into contact 3 and reflected with probability $R_3 = 1 - T_3$ into contact 2. Of interest is the correlation of currents measured at contacts 2 and 3.

rests on a number of assumptions: in addition to the white-noise limit (low frequency limit) it is assumed that the terminals are all held at a constant (time-independent) terminal-specific potential. This is possible if the mesoscopic conductor is embedded in a zero-impedance external circuit. No general statement on the sign of correlations exists if the external circuit is characterized by an arbitrary impedance.

In this work we are interested in situations for which the above mentioned proof does not apply. For instance, a voltmeter ideally has infinite impedance, and a conductor in which one of the contacts is connected to a voltmeter presents a simple example in which it is possible to measure *positive* current-current correlations [24]. In steady state transport the potential at a voltage probe floats to achieve zero net current. If the currents fluctuate the potential at the voltage probe must exhibit voltage fluctuations to maintain zero current at every instant. As has been shown by Texier and Büttiker, the fluctuating potential at a voltage probe can lead to a change in sign of a current-current correlation [24].

A voltage probe also relaxes the energy of carriers, it is a source of dissipation [25–28]. Probes which are non-dissipative are of interest as models of dephasors. At low temperatures dephasing is quasi-elastic and it is therefore reasonable to model dephasing in an energy conserving way. This can be achieved by asking that a fictitious voltage probe maintains zero current at every energy [29]. Ref. [17] presents an application of this approach to noise-correlations in chaotic cavities.

It is of interest to investigate current-correlations in the presence of such a dephasing voltage probe and to compare the result with a real dissipative voltage probe. No examples are known in which a dephasing probe leads to positive correlations. However, there exists also no proof that correlations in the presence of dephasing voltage probes are always negative.

The proof that correlations in Fermionic conductors are negative also does not apply in the high-frequency regime. We discuss the frequency-dependence of equilibrium fluctuations in a ballistic wire to demonstrate the ocurrence of positive correlations at large frequencies.

Another form of interactions which can induce positive correlations comes about if a normal conductor is coupled to a superconductor. Experiments have already probed shot noise in hybrid normal-superconducting two-terminal structures [30–34]. In the Bogoliubov de Gennes approach the superconductor creates excitations in the normal conductor which consist of correlated electron-hole pairs. The process which creates the correlation is the Andreev reflection process by which an incident electron (hole) is reflected as a hole (electron). In this picture it is the occurrence of quasi-particles of different charge which makes positive correlations possible [35–37]. The quantum statistics remains Fermi like since the field operator associated with the Bogoliubov de Gennes equations obeys the commutation rules of a Fermi field [38]. Alternatively the superconductor can be viewed as an injector of Cooper pairs [39]. In this picture is the brake-up of Cooper pairs and the (nearly) simultaneous emission of the two electrons through different contacts which makes positive correlations possible. Our discussion centers on the conditions (geometries) which are necessary for the observation of positive correlations in mesoscopic normal conductors with channel mixing. Boerlin et al. [40] have investigated the current-correlations of a normal conductor with a channel mixing central island seprated by tunnel junctions from the contacts and the superconductor. Samuelsson and Büttiker [41] consider a chaotic dot which can have completely transparent contacts or contacts with tunnel junctions. Interestingly while a chaotic cavity with perfectly transmitting normal contacts and an even wider perfect contact to the superconductor exhibits positive correlations, application of a magnetic flux of the order of one flux quantum only is sufficient to destroy the proximity effect and is sufficient in this particular geometry to change the sign of correlations from positive to negative [41]. Equally interesting is the result that a barrier at the interface to the superconductor helps to drive the correlations positive [41].

## 2. Quantum Statistics and the sign of Current-Current Correlations

In this section we elucidate the connection between statistics and current-current correlations in multiterminal mesoscopic conductors and compare them with intensity-intensity correlations of a multiterminal wave guide connected to black body radiation sources [7, 12]. We start by considering a conductor that is so small and at such a low temperature that transmission of carriers through the conductor can be treated as completely coherent. The conductor is embedded in a zero-impedance external circuit. Each contact, labeled $\alpha = 1, 2, ...$, is characterized by its Fermi distribution function $f_\alpha$. Scattering of electrons at the conductor is described by a scattering matrix $S$. The $S$-matrix relates the incoming amplitudes to the outgoing amplitudes: the element $s_{\alpha\beta,mn}(E)$ gives the amplitude of the current probability in contact $\alpha$ in channel $m$ if a carrier is injected in contact $\beta$ in channel $n$ with amplitude 1 (see [12] for a more precise definition). The modulus of an $S$-matrix element is the probability for transmission from one channel to another. We introduce a *total* transmission probability (for $\alpha \neq \beta$)

$$T_{\alpha\beta} = \mathrm{Tr}\left\{ s^{\dagger}_{\alpha\beta}(E) s_{\alpha\beta}(E) \right\} . \tag{1}$$

Here the trace is over transverse quantum channels and spin quantum numbers. This permits to write the conductance in the form [8, 12]

$$G_{\alpha\beta} = -\frac{e^2}{h} \int \mathrm{D}E\, (-df/dE) T_{\alpha\beta} . \tag{2}$$

where $f$ is the equilibrium Fermi function. The diagonal elements of the conductance matrix can be expressed with the help of $s_{\alpha\alpha}$. With the help of the *total* reflection probability $R_{\alpha\alpha} = N_\alpha - \mathrm{Tr}\left\{ s^{\dagger}_{\alpha\alpha}(E) s_{\alpha\alpha}(E) \right\}$ where $N_\alpha$ is the number of quantum channels in contact $\alpha$ we have $G_{\alpha\alpha} = e^2/h \int \mathrm{D}E\, (-df/dE)[N_\alpha - R_{\alpha\alpha}]$. Alternatively, since $\sum_\beta G_{\alpha\beta} = \sum_\alpha G_{\alpha\beta} = 0$ the diagonal elements can be obtained from the off-diagonal elements. The average currents of the conductor are determined by the transmission probabilities and the Fermi functions of the reservoir

$$I_\alpha = \frac{e}{h} \int dE [(N_\alpha - R_{\alpha\alpha}) f_\alpha - \sum_{\alpha\beta} T_{\alpha\beta}(E) f_\beta] . \tag{3}$$

In reality the currents fluctuate. The total current at a contact is thus the sum of an average current and a fluctuating current. We can express the total current in terms of a "Langevin" equation

$$I_\alpha = \frac{e}{h} \int dE [(N_\alpha - R_{\alpha\alpha}) f_\alpha - \sum_{\alpha\beta} T_{\alpha\beta}(E) f_\beta] + \delta I_\alpha . \tag{4}$$

We have to find the auto - and cross-correlations of the fluctuating currents $\delta I_\alpha$ such that at equilibrium we have a Fluctuation-Dissipation theorem and such that in the case of transport the correct non-equilibrium (shot noise) is described by the fluctuating currents. The first part of Eq. (4) represents the average current only in the case that the Fermi distributions are constant in time. This is the case if the conductor is part of a zero-impedance external circuit. If the external circuit has a finite impedance, the voltage at a contact fluctuates and consequently the distribution function of such a contact is also time-dependent. In this section we consider only the case of constant voltages in all the contacts.

We compare the current fluctuations of the electrical conductor with the intensity fluctuations of a (multi-terminal) structure for photons in which each terminal connects to a black body radiation source characterized by a Bose-Einstein distribution function $f_\alpha$. Like the electrical conductor the wave guide is similarly characterized by scattering matrices $s_{\alpha\beta}(E)$.

The noise spectrum is defined as $P_{\alpha\beta}(\omega)2\pi\delta(\omega+\omega') = \langle\delta\hat{I}_\alpha(\omega)\delta\hat{I}_\beta(\omega') + \delta\hat{I}_\beta(\omega')\delta\hat{I}_\alpha(\omega)\rangle$ with $\delta\hat{I}_\alpha(\omega) = \hat{I}_\alpha(\omega) - \langle\hat{I}_\alpha(\omega)\rangle$, where $\hat{I}_\alpha(\omega)$ is the Fourier transform of the current operator at contact $\alpha$. The zero frequency limit which will be of interest here is denoted by: $P_{\alpha\beta} \equiv P_{\alpha\beta}(\omega=0)$. The scattering approach leads to the following expression for the noise [6, 7, 12]

$$P_{\alpha\beta} = \frac{2e^2}{h}\int \mathrm{D}E \sum_{\gamma,\lambda} \mathrm{Tr}\left\{A^{\alpha}_{\gamma\lambda}A^{\beta}_{\lambda\gamma}\right\} f_\gamma(1\mp f_\lambda)\,. \tag{5}$$

The matrix $A^{\beta}_{\lambda\gamma}$ is composed of the matrix elements of the current operator in lead $\beta$ associated with the scattering states describing carriers incident from contact $\lambda$ and $\gamma$ and is given by

$$A^{\alpha}_{\gamma\lambda} = \delta_{\alpha\gamma}\delta_{\alpha\lambda} - s^{\dagger}_{\alpha\gamma}(E)s_{\alpha\lambda}(E)\,. \tag{6}$$

In Eq. (5) the upper sign refers to Fermi statistics and the lower sign to Bose statistics.

To clarify the role of statistics it is useful to split the noise spectrum in an equilibrium like part $P^{eq}_{\alpha\beta}$ and a transport part $P^{tr}_{\alpha\beta}$ such that $P_{\alpha\beta} = P^{eq}_{\alpha\beta} + P^{tr}_{\alpha\beta}$. We are interested in the correlations of the currents at two different terminals $\alpha \neq \beta$. The equilibrium part consists of Johnson-Nyquist noise contributions which can be expressed in terms of transmission probabilities only [7, 12]

$$P^{eq}_{\alpha\beta} = -\frac{2e^2}{h}\int \mathrm{D}E\,(T_{\alpha\beta}f_\beta(1\mp f_\beta) + T_{\beta\alpha}f_\alpha(1\mp f_\alpha)\,. \tag{7}$$

Since both for Fermi statistics and Bose statistics $f_\alpha(1\mp f_\alpha) = -kTdf_\alpha/dE$ is positive, the equilibrium fluctuations are *negative* independent of statistics. The transport part of the noise correlation is

$$P^{tr}_{\alpha\beta} = \mp\frac{2e^2}{h}\int \mathrm{D}E \sum_{\gamma,\lambda} \mathrm{Tr}\left\{s^{\dagger}_{\alpha\gamma}s_{\alpha\lambda}s^{\dagger}_{\beta\lambda}s_{\beta\gamma}\right\} f_\gamma f_\lambda\,. \tag{8}$$

To see that this expression is negative for Fermi statistics and positive for Bose statistics one notices that it can be brought onto he form [7, 12]

$$P^{tr}_{\alpha\beta} = \mp\frac{2e^2}{h}\int \mathrm{D}E\ \mathrm{Tr}\left\{[\sum_{\gamma} s_{\beta\gamma}s^{\dagger}_{\alpha\gamma}f_\gamma][\sum_{\lambda} s_{\alpha\lambda}s^{\dagger}_{\beta\lambda}f_\lambda]\right\}\,. \tag{9}$$

The trace now contains the product of two self-adjoint matrices. Thus the transport part of the correlation has a definite sign depending on the statistics.

It follows that current-current correlations in a normal conductor are negative due to the Fermi statistics of carriers whereas for a Bose system we have the possibility of observing positive correlations, as for instance in the optical Hanbury Brown and Twiss experiments [2, 3].

There are several important assumptions which are used to derive this result: It is assumed that the reservoirs are at a well defined chemical potential. For an electrical conductor this assumption holds only if the external circuit has zero impedance. The above considerations are also valid only in the white-noise (or zero-frequency limit). We have furthermore assumed that the conductor supports only one type of charge, electrons or holes, but not both. Below we are interested in examples in which one of these assumptions does not hold and which demonstrate that also in electrical purely normal conductors we can, under certain conditions, have positive correlations.

## 3. Coherent Current-Current Correlation

We now consider the specific conductor shown in Fig. 1. It is a schematic drawing of the conductor used in the experiment of Oberholzer et al. [21]. The sample is subject to a high magnetic field such that the only states which connect one contact to another one are edge states [42, 43]. We consider first the case when there is *only one edge state* (filling factor $\nu = 1$ away from the quantum point contacts). The edge state is partially transmitted with probability $T_1$ at the left quantum point contact and is partially transmitted with probability $T_3$ at the right quantum point contact. The potential $\mu_1 = \mu + eV$ at contact 1 is elevated in comparison with the potentials $\mu_2 = \mu_3 = \mu$ at contact 2 and 3. Thus carriers enter the conductor at contact 1 and leave the conductor through contact 2 and 3. Application of the scattering approach requires also the specification of phases. However, for the example shown here, without closed paths, the result is independent of the phase accumulated during traversal of the sample and the result can be expressed in terms of transmission probabilities only.

At zero temperature we can directly apply Eq. (9) to find the cross-correlation. Taking into account that only the energy interval between $\mu_1$ and $\mu$ is of interest we see immediately that $P_{23} = \mp \frac{2e^2}{h}|eV|[s_{31}s_{21}^{\dagger}s_{21}s_{31}^{\dagger}]$ which is equal to $P_{23} = \mp \frac{2e^2}{h}|eV|[s_{21}^{\dagger}s_{21}s_{31}^{\dagger}s_{31}]$. But $s_{21}^{\dagger}s_{21} = T_1 R_3$, where $R_3 = 1 - T_3$ and $s_{31}^{\dagger}s_{31} = T_1 T_3$ and thus

$$P_{23} = -\frac{2e^2}{h}|eV|T_1^2 R_3 T_3 \,. \tag{10}$$

Transmission through the first quantum point contact thins out the occupation in the transmitted edge state. This edge state has now an effective distribution $f_{eff} = T_1$. The correlation function has thus the form $P_{23} = -\frac{2e^2}{h}|eV|f_{eff}^2 R_3 T_3$.

For $T_1 = 1$ we have a completely occupied beam of carriers incident on the second quantum point contact and the correlation is maximally negative with $P_{23} = -\frac{2e^2}{h}|eV|R_3T_3$. In this case the correlation is completely determined by current conservation: Denoting the current fluctuations at contact $\alpha$ by $\delta I_\alpha$ we have $\delta I_1 + \delta I_2 + \delta I_3 = 0$. Consequently since the incident electron stream is noiseless $\delta I_1 = 0$ we have $P_{23} = -P_{22} = -P_{33}$. Therefore if the first quantum point is open the weighted correlation $p_{23} = P_{23}/(P_{22}P_{33})^{1/2} = -1$. The fact that an electron reservoir is noiseless is an important property of a source with Fermi-Dirac statistics [20].

If the transmission through the first quantum point contact is less than one the diminished occupation of the incident carrier beam reduces the correlation. Eventually in the non-degenerate limit $f_{eff}$ becomes negligibly small and the correlation between the transmitted and reflected current tends to zero. This is the limit of Maxwell-Boltzmann statistics.

The experiment by Oberholzer et al. [21] measured the correlation for the entire range of occupation of the incident beam and thus illustrates the full transition from Fermi statistics to Maxwell-Boltzmann statistics. The experiment by Oliver et al. [19] even though it is for a different geometry (and at zero magnetic field) is discussed by the authors in terms of the same formula Eq. (10). The range over which the contact which determines the filling of the incident carrier stream can be varied is, however, more limited than in the experiment by Oberholzer et al..

Before continuing we mention for completeness also the auto-correlations

$$P_{33} = \frac{2e^2}{h}|eV|T_3T_1(1 - T_3T_1)\,, \tag{11}$$

$$P_{22} = \frac{2e^2}{h}|eV|T_1R_3(1 - T_1R_3)\,, \tag{12}$$

For $T_1 = 1$ this is the partition noise of a quantum point contact [44, 45].

We are now interested in the following question: Carriers along the upper edge of the conductor have to traverse a long distance from quantum point contact 1 to quantum point contact 3 (see Fig. 1). How would quasi-elastic scattering (dephasing) or inelastic scattering affect the cross correlation Eq. (10)? For the case treated above where only one edge state or a spin degenerate edge is involved the answer is simple: the cross correlation remains unaffected by either quasi-elastic or inelastic scattering. The question (asked by B. van Wees) becomes interesting if there are two or more edge states involved. It is for this reason that Fig. (1) shows two edge channels.

## 4. Cross correlation in the presence of quasi-elastic scattering

Incoherence can be introduced into the coherent scattering approach to electrical conduction with the help of fictitious voltage probes. (see Fig. 3). Ideally a voltage

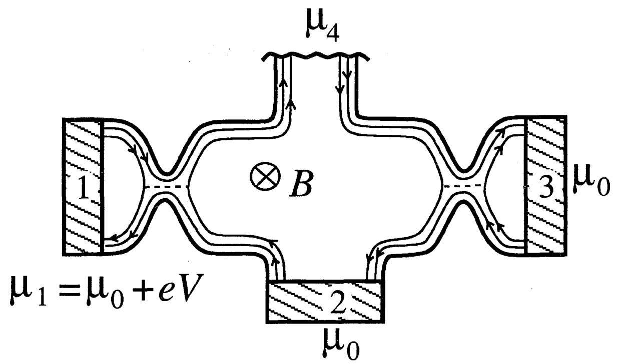


*Figure 2.* A voltage probe at the upper edge generates inelastic scattering or dephasing depending on whether the total instantaneous current or additionally the current at every energy is set to zero. After Texier and Büttiker [24].

probe maintains zero net current at every instant of time. [ A realistic voltmeter will have a finite response time. However since we are concerned with the low-frequency limit this is of no interest here.] A carrier entering a voltage probe will thus be replaced by a carrier entering the conductor from the voltage probe. Outgoing and incoming carriers are unrelated in phase and thus a voltage probe is a source of decoherence. A real voltage probe is dissipative. If we wish to model dephasing which at low temperatures is due to quasi-elastic scattering we have to invent a voltage probe which preserves energy. de Jong and Beenakker [29] proposed that the probe keeps not only the total current zero but that the current in each energy interval is zero at every instant of time. Noise correlations in the presence of a dephasing voltage probe have been investigated by van Langen and the author for multi-terminal chaotic cavities [17].

In the discussion that follows we will assume, as shown in Fig. 1 that the outer edge channel is perfectly transmitted at both quantum point contacts. Only the inner edge channel is as above transmitted with probability $T_1$ at the first quantum point contact and with probability $T_3$ at the second quantum point contact. Elastic inter-edge channel scattering is very small as demonstrated in experiments by van Wees et al. [46], Komiyama et al. [47], Alphenaar et al. [48] and Mueller et al. [49] and below we will not address its effect on the cross correlation. For a discussion of elastic interedge scattering in this geometry the reader is referred to the work by Texier and Büttiker [24]. We wish to focus on the effects of quasi-elastic scattering and inelastic scattering. The addition of the outer edge channel has no effect on the noise in a purely quantum coherent conductor. Edge channels with perfect transmission are noiseless [6].

To model quasi-elastic scattering along the upper edge of the conductor we