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Continued after Index

Elaheh Kheirandish

The Arabic Version
of Euclid's *Optics*

(*Kitāb Uqlīdis fī Ikhtilāf al-manāẓir*)

Edited and Translated with
Historical Introduction
and Commentary

Volume I

With 73 Illustrations



Springer

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To my parents
Asadollah and Parvin-Dokht
(the “Lion of God” and the “Daughter of Pleiades”)

آن قوم که راه بین فتادند وشدند
کس را بیقین خبر ندادند وشدند
آن عقده که هیچکس نتانست گشاد
هریک گرهی برآن نهادند وشدند

منسوب به خواجه نصیر الدین طوسی
(نقل از احوال و آثار مدرس رضوی، ص ۶۱)

That group of people who, as guides, came and passed
and informed no one with certitude, as they passed
into that complex tangle, unraveled by anyone
each came to place a knot, and so they passed

Attributed to Naṣīr al-Dīn al-Ṭūsī (d. 672/1274)
(Persian poem quoted from Mudarris-i Raḡavī, *Aḥvāl va Āthār*, p. 61)

Preface

The two volumes of *The Arabic Version of Euclid's Optics* document a rare and complex history in which the translation of a single text into another language has had consequences for the transformation of the field far beyond the literal meaning implied by the title. In the fascinating account of the transition of the field from Greek *optika* to Arabic *al-manāẓir*, it was not only *what* was—or was not—transmitted that directed the course of developments; it was also *how* things were transmitted. In the case of the *Optika* of Euclid (ca. 300 B.C.), the first surviving work devoted entirely to the subject, its text was not merely transmitted to the Islamic world (and transmitted more fully and widely than the much more comprehensive work of Ptolemy composed in the 2nd century A.D. with the same title). Its initial transmission from Greek into Arabic was such as to involve textual transformations in some critical points. Not only were some central Greek terms and expressions transformed when turned into Arabic “equivalents.” The passage of the most central concepts of the Greek optical tradition through a chain of flexible terminologies and interchangeable orthographies in *both* Greek and Arabic was to have notable consequences for further developments. With the publication of *The Optics* of Ibn al-Haytham (5th/11th century), itself unique in the almost miraculous share it took from the riches of Euclidean *and* Ptolemaic optics, such consequences can be readily examined in the case of what is by far the most important development of all. And, just as in the case of Ibn al-Haytham’s *Kitāb al-Manāẓir*, the transmission of Euclid’s *Kitāb al-Manāẓir* involved both “fortunes” and “misfortunes”—to quote A.I. Sabra’s preface—from the standpoint of later developments. The transformations together with their consequences form the main core of the two volumes (Volume I containing the Arabic text and English translation, Volume II the textual and historical commentary), in the hope of capturing a reflection of the complex process of scientific development itself.

To speak of “Euclid’s *Optics*” and its Arabic “version” or of “the translation of a single text into another language” would be barely meaningful with the continuing problems of textual attribution and identification in the Greek tradition, had it not been that, from the standpoint of the Arabic tradition, there *was* such a text as the “translation” of “Euclid’s book on *Optics*.”

From our standpoint, and perhaps that of the Greek tradition itself, “Euclid’s *Optics*” has never been fully meaningful as a single point of reference. For a long time, it referred to a text with such a title in various early editions (Zamberti, 1505, Pena, 1557, David Gregory, 1703, to name but a few). About a century ago, it came to refer to *Heiberg’s Euclid*, a different text based on the manuscripts of a Greek *Optika* that J.L. Heiberg edited as *Euclidis Optica*, as distinguished from the previously known set of Greek manuscripts now presented in the same edition with the title *Opticorum recensio Theonis*, as a recension by the 4th-century commentator, Theon of Alexandria (Heiberg, 1895). In the wake of the recent reexaminations of the Greek Euclidean tradition by Alexander Jones and Wilbur Knorr (in what has left less and less of Euclid in the former and more and more of it in the latter), there is now also *Euclid’s Optics*, according to *their* respective standpoints.

Euclid’s *Optics* of the Arabic tradition *did* represent a fixed identity, the name of Euclid being repeatedly attached to the Arabic versions passing under the title of “*Optics*”; and it was a text that, even as the product of a textual transmission involving more than one version, was passed on as a single voice. As the Arabic was *not*, in any of its close variations, exactly what was in either of the Greek versions associated with Euclid’s name, and was in fact quite transformed when it came to the text’s most fundamental assumptions, the “translation” was at once the Arabic version of the Euclidean text and the Arabic “version” of the Euclidean visual theory.

The challenging study of such a tradition has come to involve much more than what is to be expected even in the most straightforward textual treatments of this kind. With any study whose main sources are largely in manuscript form, various problems with extant copies, from identification to actual access, are to be reasonably anticipated. In this case, even establishing their mere existence—let alone current whereabouts—has been a problem, at times requiring persistent investigative (and occasionally detective) work. It so happens that, of the few extant manuscript copies of the main text, one has traveled apparently from Cairo to Tehran, Mashhad, New York, Paris, Basel, and on to unknown destinations, half of this in the course of the last decade; three have been housed between Istanbul and Cairo, both places with a long (and, to some, familiar) history of limited accessibility; and one is supposed to be still sitting in war-zone Kabul, with none of the request letters (even to presidents! as recommended by Afghani colleagues) bearing any fruit. The stories of the trace (or rather chase) are not as colorful in the case of the other *Manāẓir* texts, except for occasions to examine heavily guarded illuminated manuscripts in the museums of Istanbul and dust-covered codices in private libraries in Qum. But the attempts to improve the problematic documentation status of the *Taḥrīr al-Manāẓir* of Naṣīr al-Dīn al-Ṭūsī (d. 672/1274), often confused with the early Arabic version, and its short attachment *Iṣlāḥ al-Manāẓir*, occasionally taken for the *De aspectibus* of al-Kindī (d. ca. 257/870), did result in the identification of over two-thirds of their surviving manuscript copies, many being unreported (and a few misrepresented) in the most widely consulted of reference sources.

For the completion of the work, I feel privileged to have had timely support from many sources and in many forms. First and foremost, I greatly value the fortune of having received the inspiring guidance of Professor A.I. Sabra at a critical intellectual moment and throughout the course of a study that began life as a dissertation project and reached its current state under his genuine “supervision.” An award from the National Science Foundation allowed for the initial identification and collection of manuscript sources included in the study and was particularly crucial for personal examination of those that were either difficult to obtain or, in the most critical cases, difficult to read in the best reproduced form. Subsequent appointments as a postdoctoral fellow at the Department of the History of Science at Harvard University, as a member of the Research Faculty of History and Philosophy of Science at Tehran’s Institute for Humanities and Cultural Studies, and, most recently, as a Resident Fellow of the Dibner Institute for the History of Science and Technology at MIT have provided valuable opportunities and facilities for the necessary revisions and updates reflected in this publication.

The occasion also calls for grateful acknowledgment of the helpful assistance of several libraries in the Middle East and Europe that hold relevant manuscripts (itemized in Lists I and II, v. 1, pp. xxxiv–liii): the *Topkapı*, Süleymaniye, and Köprülü Libraries of Istanbul, University Library of Leiden, Bibliothèque Nationale of Paris, India Office of London, Bodleian Library of Oxford, John Rylands Library of Manchester, and Medicea-Laurenziana Library of Florence. As for largely unexplored manuscripts existing in collections throughout Iran (some in multiple copies or with the oldest transcription dates), individual acknowledgments are particularly due for the kind cooperation of the staff at Tehran University Markazī (Central) Library, Majlis-i Shawrā, Muṭahharī (Sipahsālār), and Malik Libraries in Tehran, Āstān-i Quds Library in Mashhad, Mufattiḥ (Pārs) Library in Shiraz, the Libraries of Isfahan and Shiraz University, the supplementations provided by the private libraries of Ustād ‘Ali-Naqī Munzavī in Tehran and Ayatollah Ḥasan-zādiḥ-i Āmuli in Qum (the Library of Ayatollah Mar‘ashī-yi Najafī in Qum did not have facilities for women at the time of request), and, last but not least, the personal and financial support of my father, Asadollah Kheirandish, who was once my only local source of contact with these libraries. The intervention of a few other people made other important manuscripts available to me. David Partington, the former director of the Middle East Division of Widener Library, and Richard Lorch, at the University of Munich, acted as crucial links for my eventual access to a total of three manuscript copies of the main text. Professor Everett Rowson and, more recently, Skuli Sigurdsson kindly seconded my efforts in obtaining films of related manuscripts from Cairo and Krakow; and Professor Charles Burnett was instrumental in my last-minute access to the photograph of the manuscript folio included in the second volume.

Expressions of gratitude must extend to other aspects of the study. Professor John Murdoch first woke in me an appreciation for Greek and Latin intellectual history and awareness of problems associated with each. Colleagues Adam Schulman, William Newman, Dana Miller, and Tzvi Langermann offered

insights from their respective fields of specialty on some complexities in the Greek and Latin texts and on the question of Syriac or Hebrew links. Professors Dimitri Gutas and Paul Kunitzsch generously shared their expert knowledge on the question of the exact name and identity of the Arabic translator and a few other specific problems. And Professors Jamil Ragep and David King took me in their own ways far beyond my expectations in improving the microscopic details of related work. Special thanks naturally goes to Professor Gerald J. Toomer for his careful and critical review of various stages of the entire work and his crucial role in having it published. It is unfortunate that the related work of Professor Roshdi Rashed remained unavailable to me until after I had submitted my manuscript for publication.

On the technical aspects of the work, the fonts for the Greek and Arabic transcriptions and English transliteration were designed (and in some cases tailor-made) by Professor Toomer, and those for Persian by Professor Wheeler Thackston. The attempted upgrades in the photographic quality of the manuscript plates carry the expert touch of Kamran Adle. And the technical support of my *truly better* half, Hormoz Goodarzy, has been indispensable throughout the work, in crisis and in calm, in substance and in form. For his caring support throughout the long course of the study itself, it is hard to find the right words.

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Volume I

Part I

Introduction to the Texts

1 The Arabic Versions

The corpus of sources for the study of the Arabic tradition of Euclidean optics is formed by a few genres of works. First there are the founding texts of the tradition, namely the Arabic versions of the Euclidean text, representing a major part of the original Greek as found in Heiberg's *Euclidis Optica*.¹ Then there is a set of dependent texts, which together with the Arabic versions of the text, form the tradition as it developed in the four centuries between and occasionally beyond the Arabic versions of the 3rd/9th and 7th/13th centuries. And finally there is a series of related texts, reflective of or responsive to different aspects of Euclidean optics, through which the tradition may be further studied or its chronological and geographical boundaries further extended.

The Arabic versions of the Euclidean text also survive in several forms. The earliest form is best represented by the odd-numbered set of manuscripts: m1, m3, m5 [see Description and Analysis of Manuscripts, and List of Manuscripts, hereafter Description and List, pp. xxvi–xxvii and p. xxxiv, respectively]. Characterized by the title *Ikhtilāf al-manāẓir*, this can be considered an early translation. It is a “translation” (*tarjama*) according to the colophon of one of the manuscript copies in this same set (m5), and it is early because of certain considerations including the dates of its supposed translator. The name to which the translation is attributed, Hiliyā ibn Sarjūn [reading for Hlil ibn Sarhūn, see Manuscript Plate V, p. 240, and The Arabic “Version,” v. 2, p. xxii, n. 19], can be in fact associated with more than one early translation from Greek into Arabic: a translation, together with al-Hajjāj ibn Yūsuf ibn Maṭar (fl. ca. 170/786–218/833), of Ptolemy's *Almagest* (ca. 212/827–828), a text for which the *Optics*, being one of the “Intermediate Books” (*Kutub al-Mutawassiṭāt*), was to act as preparatory;² and with slight variation of the name, also of Cassianus Bassus's *Geoponika* [see The Arabic “Version,” v. 2, p. xxii, n. 20–21]. If not the man himself, then he may be the father or the son [ibid., n. 22–23].

¹ Greek text ed. Heiberg, *Euclidis Optica, Opticorum recensio Theonis, Catoptrica, cum scholiis antiquis* in *Euclidis Opera Omnia*, eds. Heiberg and Menge, v. 7: *Euclidis Optica*, pp. 1–121; *Opticorum recensio Theonis*, pp. 143–247. English translation, Burton, “The Optics of Euclid”; French translation, Ver Eecke, *Euclide, l'Optique et la Catoptrique*. For a reexamination of Heiberg's position, including the attribution of the two Greek versions to Euclid and Theon of Alexandria, see Jones, “Peripatetic and Euclidean Theories,” and Knorr, “Pseudo-Euclidean Reflections in Ancient Optics.” See also Textual Transmission, p. lxi, and The Arabic “Version,” v. 2, pp. xix–xx and n. 7.

² Also known as the “Middle Books” or the “Little Astronomy,” this is the collective name for mathematical works to be studied intermediately between Euclid's *Elements* and Ptolemy's *Almagest*. They consist of a number of Greek texts including the *Optika*, *Data*, and *Phaenomena* by Euclid, *Spherics* of Theodosius and of Menelaus, and the *Moving Sphere* of Autolycus; see Steinschneider, “Die mittleren Bücher.”

Whether or not the translation of the *Optics* was – as were the Arabic *Elements* or *Almagest*, the two masterpieces to be studied before and after it –³ carried out during the reign of the ‘Abbasid Caliph al-Ma’mūn (r. 198–218/813–833),⁴ what circulated under the title of *Ikhtilāf al-manāẓir* must be placed among the oldest compositions of the period on independent evidence. It is with this same title that the expression “old translation” (*al-naql al-qadīm*), a common designation for the earliest phases of the 2nd/8th–3rd/9th-century translation movement, is repeatedly associated [see Summary and Commentary: Title, v. 2, p. 2], and there are references to a text with such a title starting from at least the middle of the 3rd/9th century.⁵

A slight variation to this early Arabic version is represented by the even-numbered set of manuscripts, m2 and m4, with titles representing the short form *al-Manāẓir* [see Description, pp. xxvi–xxvii and List I, p. xxxiv], and an apparently more limited circulation. The variance with the former set is most striking in the case of the very first Euclidean definition, as in the case of the two main extant Greek versions identified and edited as the respective texts of Euclid and Theon of Alexandria about a century ago, and versions A and B as reexamined and renamed recently [see Textual Transmission, p. lxi, and n. 51].

³ The explicit statement that the “Intermediate Books” (*al-Kutub al-Mutawassiṭāt*) were to be studied after Euclid’s *Elements* and before Ptolemy’s *Almagest* is found in some manuscripts, for example the Arabic translation of Euclid’s *Elements* [MS Oxford, Bodleian Library, Thurston 11, fol. 213a]: “*dhikr al-kutub alladhī yuḥtāju ilā qirā’atihā qabli Kitāb al-Magistī wa tu’rifu bi al-Mutawassiṭāt*” (“citation of the books needed to be read before the *Almagest* and known as the Intermediate [Work]s”). There, *Kitāb Ikhtilāf al-manāẓir li-Uqlīdis* appears as the second title in the list [ibid., fol. 213b], but the order of the “Middle Texts” within the mathematics curriculum is rather flexible for both the early version and its later recension.

⁴ On the Arabic tradition of Euclid’s *Elements*, see Murdoch, “Euclid,” *DSB*, 4: pp. 438–443, and De Young, “The Arabic Textual Traditions of Euclid’s *Elements*.” For the case of Ptolemy’s *Almagest*, see references to the standard literature in Toomer, “Ptolemy,” *DSB*, 11, p. 203, and Kunitzsch, *Der Almagest*, pp. 60–71. On the question of dating, see The Arabic “Version,” v. 2, n. 23–24.

⁵ The reference “*qāla Uqlīdis fī Kitāb Ikhtilāf al-manāẓir*” (“Euclid said in the *Optics*”) appears in an apparently early work [before 250/864?, according to Krause, “Stambuler Handschriften,” p. 513] by Aḥmad ibn ‘Īsā, entitled *Kitāb al-Manāẓir wa al-marāyā al-muḥriqa*; see Ḥ in List II, p. xlv (ḥ1, fol. 89a); see also The Arabic Tradition, p. liv, and Summary and Commentary: Proposition 10, v. 2, pp. 47–48, n. 182–183. There is also al-Kindī’s (d. ca. 257/870) *De aspectibus [diversitatum aspectus (Ikhtilāf al-manāẓir?)]*; see Björnbo and Vogl, *Alkindi, Tideus und Pseudo-Euklid*, pp. 1–41] based on it [see Summary and Commentary: Definition 1, v. 2, p. 12], though not preserved in Arabic for close textual comparison. Such a text by Euclid is also reported in al-Ya’qūbī’s (d. 284/897) *Ta’riḫ* (ca. 267/880), ed. Houtsma, part I, p. 139. The dates of al-Ya’qūbī and Ibn ‘Īsā are from Krause, “Stambuler Handschriften,” p. 441, n. 4, and p. 513, and Brockelmann, “al-Ya’kūbī,” *ET*¹.

The even-numbered manuscripts do have an unmistakable trace of the opening definitions of Heiberg's *Recensio Theonis* [or version B], just as the odd-numbered ones do in other parts of the text [see Summary and Commentary: Proposition 35, v. 2, p. 74]; but none have a preface or a consistent correspondence, starting from the very next definition, to be considered its direct translation.⁶ If, on the other hand, an early revision of the Arabic *Optics* had ever been prepared, as in the case of other Greek scientific works,⁷ then a better candidate may be the even-numbered set, representing slight and at times critical divergence, still in the name of Euclid.⁸ There is, however, no evidence for the existence of such a "revision." The text *Iṣlāḥ al-Manāẓir*, attributed to al-Kindi (d. ca. 257/870) [§, see List I, p. xxxv], is, in the form in which it has reached us, a "correction" (*iṣlāḥ*) of two problematic Euclidean propositions [see Edition and Translation, pp. 226–229], not a "revision" (*taṣḥīḥ*) as such.⁹ Neither is there an indication that names ordinarily associated with such "revisions" [Ishāq ibn Ḥunayn (d. ca. 298/910), Thābit ibn Qurra (d. ca. 289/901)] or other names linked with this particular translation [Ḥunayn ibn Ishāq (d. ca. 264/877) and Qusṭā ibn Lūqā (d. ca. 300/912)] were ever involved in such an effort [see The Arabic "Version," v. 2, p. xxii]. The existence of a revision/restoration effort, however, can be well supported, and the possibility of a prototype to which members of one or both of the above sets may be related cannot be ruled out.¹⁰

⁶ There is no extant text documented as the Arabic version of that particular Greek text, nor is there any reference to Theon's name in that connection. But in an unpublished study of the medieval Euclidean tradition [Theisen, *The Mediaeval Tradition*, p. 324, n. 10], it is suggested that the Vatican manuscript Urbina 1329, with 64 propositions [and no preface], "is probably a Latin translation of an Arabic version of Theon's *Recension*."

⁷ On the case of the *Elements*, see Murdoch, *Euclid*, p. 443; for the *Almagest*, see Kunitzsch, *Der Almagest*, p. 67.

⁸ The colophon of m2 contains a reference to Euclid's name; for the details of the two variants, including stylistic differences, see Description, pp. xxvii–xxix. For discussions of authorship and of textual language, see The Arabic "Version," v. 2, p. xxv–xxviii.

⁹ The treatment of the two propositions in *Iṣlāḥ al-Manāẓir* does not match those in the Euclidean text [Propositions 60–61], to the numbers of which specific reference is made, such that § could be considered an excerpt from the revision of M in those parts. See respective editions, pp. 226–229 and pp. 212–215, and Summary and Commentary: Propositions 60–61, v. 2, pp. 95–97. Neither does § overlap with al-Kindi's *De aspectibus* or match Ṭūsī's *Tahrir* of those same propositions, where such corrections usually follow Euclidean treatments, thereby making such a sequel by Ṭūsī unnecessary.

¹⁰ The restoration assumption is supported by the missing first page of one of the manuscripts (m3), the testimony of one from the other variant set (m2) on the corrupt nature of the latter's original, and the striking manuscript variance of the opening lines. For similar arguments for the same part of the Greek, see Summary and Commentary: Definitions, v. 2, p. 6, n. 60. The prototype assumption, which is still reconcilable with restoration efforts, may be entertained in the light of the title and description in al-Ya'qūbi's *Ta'rikh* (ca. 267/880) not being identical to any of the extant copies m1–m5; see Supplementary Passages, p. 243, and Variation Sample 1, v. 2, p. xxxviii. The

The two early variations [i.e., m1, m3, m5 and m2, m4] may be different enough [see Description, p. xxviii] to suggest a revision or restoration effort at work, but not enough to be considered as two distinct versions. They are not even different consistently enough [see Summary and Commentary: Definition 1, 2; Proposition 35, v. 2, p. 7, 17, and 74 respectively] to be respectively assigned to the traditions of the two main Greek versions. With the largely uniform content and structure of all manuscript copies, and the explicit association of members of *both* the odd- and even-numbered manuscript sets with the name of Euclid [m1, m2, m3, and m5], they have been edited here as one text [hereafter M, see Edition and Translation, pp. 2–225]. Variant readings appear in the critical apparatus below the Arabic text and in critical cases alongside the Greek separately [see Variation Samples 1–4, v. 2, pp. xxxvii–xlv]. Textual variance of all definitions and propositions with reference to Greek is treated in the corresponding Summary and Commentary, v. 2, pp. 1–99.

In addition to the “early” Arabic versions – the old translation [as early as at least ca. 212/827–828 on account of the dates of the supposed translator] and its slight variation [as early as before ca. 257/870 on account of al-Kindī’s possible response to it, see Summary and Commentary: Definition 1, v. 2, p. 12] – there is a much later recension of the same 64 propositions with the title of *Tahrīr al-Manāẓir* [T, see List I, p. xxxvii]. This can be distinguished from the earlier versions, most immediately by its extended definitions and condensed demonstrations. Composed by Naṣīr al-Dīn al-Ṭūsī (d. 672/1274), as part of his *Tahrīr al-Mutawassiṭāt* (*Recension of the Intermediate Works*),¹¹ and completed in the year 651H (=1253) as reported in a number of its many manuscript copies, this late version has a well-known author and firmly established date, unlike the earlier versions. The contrast is to include the “Correction” (*Iṣlāḥ*) which though ascribed to al-Kindī in all of its identified manuscript copies, having been a companion piece to Ṭūsī’s *Tahrīr al-Manāẓir* in all but one case, has been once even labeled “among the notifications of the *Khwāja*” [namely Ṭūsī] himself.¹²

alternative vocabulary of the overlapping part of the Arabic text of the pseudo-Euclidean *De speculis*, suggesting an early and unsettled language [see Supplementary Passages, p. 243 and The Arabic “Version,” v. 2, p. xxvi, n. 28], and comparable formulations of the same [see Supplementary Passages, pp. 244–245] may be used to support either or both assumptions [see Summary and Commentary: Definitions, v. 2, pp. 7–8, n. 61–63].

¹¹ Published text, Tehran lithograph edition, 1304 (=1886-87): 54–75; Hyderabad, 1358 (=1939): 2-24; *Revue de l’Institut des Manuscrits Arabes (RIMA)* 1383=1963, 9: 251-290.

¹² Of the extant copies (§1-§23), one in Isfahan (§9) has the title “*Min ifādāt al-Khwāja*” (“Among the notifications of the *Khwāja*”) [a common title for Ṭūsī]. All copies but the one in Paris (§8), which lacks the preceding *Tahrīr*, are cited in List I with reference to the entry under T. On the plausibility of such an early correction of the two corrupt Greek propositions, see Summary and Commentary: Propositions 60–61, v. 2, pp. 95–97.

There is at least one more text, *Tajrīd al-Manāẓir* [J in List I, p. xlv], composed by a contemporary of Ṭūsī named Ibn Abī Jarāda (ca. 676/1277),¹³ that must be considered as belonging directly to this tradition. And there are a number of others in whose titles the appearance of the term *al-Manāẓir* at once reveals their close association with this tradition. These are introduced separately [see The Arabic Tradition, p. liv], and their manuscripts are listed individually [under Supplementary Texts; see List II, p. xlv].

The main sources for the study of the Arabic tradition of Euclid's *Optics* are still largely in manuscript form, and their documentation status has long remained incomplete at best, misleading at worst. For the early Arabic version, M, six extant manuscript copies are cited [four in Sezgin, *GAS V*, p. 117, two others in King, "Notes on Sources," p. 453; King, *Cairo Catalogue*, v. 2, p. 1029; and King, *Cairo Survey*, p. 24], of which at least two [from the first set] are not to be found as reported. For Ṭūsī's later recension, T, the total reported cases are close to a third of the existing ones [Sezgin, *GAS V*, p. 117, lists 18 out of at least 58; Krause, "Stambuler Handschriften," p. 500, contains one addition; King, "Notes on Sources," p. 453; King, *Cairo Catalogue*, v. 2, p. 1029; King, *Cairo Survey*, p. 24, adds four others]; in the case of the published copies of the same work, only two out of three are usually mentioned. The earliest one, Tehran, 1304 (=1886–1887) [see t48 in List I, p. xliiii] is to my knowledge cited, with one exception [Storey, *Persian Literature*, v. 2, p. 2], in only a few sources in Persian. The report of a Persian translation of *al-Manāẓir* [Dorn, *Chanykov*, p. 260; Steinschneider, "Euklid bei den Arabern," p. 101; Theisen, *The Mediaeval Tradition*, p. 10; Storey, *Persian Literature*, v. 2, part 3, p. 455: "doubtless of Ṭūsī's Arabic version"] seems to be part of a recurring confusion with a Persian *al-Masākin*.¹⁴ In the more dramatic case of the supplementary correction §, not only is it mixed up with al-Kindī's *De aspectibus* [see repeated reference to a Latin translation: Carmody, *Arabic Astronomical Sciences*, p. 79; Sezgin, *GAS V*, p. 117] and largely overlooked even in catalogues listing its companion text T;¹⁵ of at least 24 extant manuscript copies of it [see List I, pp. xxxv–xxxvii and p. liii], a total of three have been so far acknowledged [Brockelmann, *GALS I*, p. 374, and Sezgin, *GAS V*, p. 117, cite the same one; King, "Notes on Sources," p. 453; King, *Cairo Catalogue*, v. 2, p. 1029; and King, *Cairo Survey*, p. 24, two

¹³ The *Tajrīd* of Ibn Abī Jarāda survives in an apparently unique manuscript copy in Cairo's Dār al-kutub, first reported by David King; see King, "Notes on Sources," p. 453; King, *Cairo Catalogue*, p. 1030; King, *Cairo Survey*, p. 24. On the author and the slight variation in the transliteration of his name, see The Arabic Tradition, p. lx.

¹⁴ Chanykov 146, entitled *Kitāb-i Manāẓir-i Uqlīdis*, contains the text *Kitāb al-Masākin*. On other confusions between the *Mutawassīṭāt*, see The Arabic "Version," v. 2, p. xxi.

¹⁵ The independent text §8 and two of the attached copies (§14, §17) are usually the only ones cited; see List I, pp. xxxv–xxxvi. I am grateful to Jamil Ragep for supplying the requested information on manuscripts §12 and §13 while in Istanbul and attempting the same for the Atif Collection (1712, 1716), which remained inaccessible through his stay.